STOCHASTIC KAEHLER GEOMETRY

TURGAY BAYRAKTAR

Zero distribution of polynomials of high degree with random coefficients naturally appear, among others, in the context of quantum chaotic dynamics. A classical result asserts that if the coefficients are independent Gaussian random variables then zeros of random polynomials tend to accumulate near the unit circle on the complex plane. On the other hand, zeros of SU(2) polynomials are uniformly distributed on the Riemann sphere. While these results are consistent with Random Matrix Theory predictions they provide a new inside for the problem of quantum ergodicity. There are also higher dimensional generalizations of these results which form a relatively new field called Stochastic Kaehler Geometry.

Lecture 1. *From Random Polynomials to Kähler Geometry.*

In the first lecture, we will review some recent results on zero distribution of random polynomials and their generalization to compact Kähler manifolds. The main theme will be the universality of statistics of zeros of (generalized) random polynomials.

Lecture 2. *Quantum Ergodic Sequences on Riemann Surfaces.*

In the first part of this lecture, after introducing the necessary background, we will report advances on the Quantum Unique Ergodicity conjecture concerning the distribution of large frequency eigenfunctions of the Laplacian on a negatively curved manifold. By replacing Laplace eigenfunctions with modular forms one is lead to Arithmetic Quantum Ergodicity conjecture concerned with the behavior of Hecke modular forms. In the second part, we will generalize the definition of quantum ergodicity for sequences of holomorphic sections of a positive line bundle on a Riemann surface of constant curvature. Furthermore, we consider random linear combinations of such eigenfunctions and explain that almost surely asymptotic zero-distribution of random sequences behave like that of eigenfunctions.

Lecture 3. *Random Sparse Polynomials.*

The last lecture of this series, will focus on zero distribution of random Laurent polynomials. It is well-known that Newton polytope of a Laurent polynomial $f \in \mathbb{C}[z_1^{\pm 1}, \dots, z_m^{\pm 1}]$ has a crucial influence on its value distribution and in particular on its zero set. A classical result due to Bernstein-Kouchnirenko asserts that for systems $(f_1, \dots, f_m)$ of Laurent polynomials in general position the number of simultaneous zeros in the torus $(\mathbb{C}^*)^m$ depends only on their Newton polytopes. In this lecture, we will present a quantitative localization of Bernstein-Kouchnirenko Theorem. If time permits, we will also mention some applications regarding the asymptotic distribution of amoebas associated with Laurent polynomials.

Faculty of Engineering and Natural Sciences, Sabanci University