Optimization of Wave-Induced Motion of Ramp-Interconnected Craft for Cargo Transfer

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Abstract

Cargo transfer between two vessels at sea requires the ramp connection between the vessels to be as stable as possible. The complex nature of the system makes employing control methods difficult. This paper presents the results of several techniques aimed to exploit dynamics of the system to reduce the relative movement of the adjoining ramp and the two vessels. The methods include extremum seeking designed to optimize wave heading angle and ramp length, fin control to alter vessel heave to reduce pitch, and passive techniques that investigate the effectiveness of spring-damper systems at the joints of the ramp. We implement these methods on a SimMechanics model created to simulate the vessels in sea state four. We compare the simulation results from the SimMechanics model to the solutions of the equations of motion we derive using Lagrangian mechanics that show continuity in the results. The results of the simulations with control algorithms implemented show the successful reduction of the pitch angle in the ramp. We provide the results of these simulations as well as a detailed account of how each aspect of the system was modeled.

1. INTRODUCTION

The transfer of cargo over a ramp from a LMSR (large, medium-speed, roll-on/roll-off) vessel to a connector vessel in high sea states represents significant challenges for ship and control system designers. The goal of this project is to determine the actuation/sensing requirements and to devise control and real-time optimization algorithms for reducing the oscillations of the pitch, roll, and yaw angles between the ramp and each vessel.

We investigate a system that consists of a Sea Base (the large vessel) and a T-Craft (the connector vessel) connected by a ramp that can vary in length. Due to the number of degrees of freedom (DOF) involved in this system, deriving the equations of motion is challenging, even when the hydrodynamic force laws are known. As an initial effort, we create a SimMechanics model to simulate the system and implement the control algorithms. The vessels were modeled as half cylinders and the ramp as a rectangular solid as shown in Figure 1. Following this, we derive the equations of motion for the system using a Lagrangian mechanics approach. However, the SimMechanics model proves to be a simpler way to implement control and optimization techniques without re-deriving the equations of motion. The SimMechanics model consists of a series of rigid bodies acted on by a wave force model that is elaborated on below. The solution to equations of motion provides a cross reference with the results of the rigid body simulations which validate each other. Three joint cases connecting each ship to the ramp are explored: Pitch only, Pitch-Roll Joint, and Pitch-Roll-Yaw Joint. These various joint models influence the angle oscillations per DOF being considered. Extremum seeking is used to identify the optimal ramp length and ocean wave incidence from various initial positions to minimize pitch angle amplitude. The goal is to employ this method in sea states where the optimal configuration is unknown. Passive control techniques are modeled after an automotive shock absorber and are employed at the joints as an effort to reduce the angle amplitudes for the various DOFs in the different joint cases. Finally, an active control approach is tested to control rotating fins in order to create variable lift and reduce the relative heave of the vessels while connected in the pitch only joint case.

In all joint cases, simulations are conducted with and without control actuation to test the effectiveness of each algorithm. The results are compared against each other to determine the effectiveness of each technique. A full range of results as well as explanation into each setup are explained in further detail.

2. MODELING WAVES AND SHIP MOTIONS

2.1. Modeling wave forces

Environmental disturbances for ship modeling are considered as waves, wind and ocean currents. In this paper, direct effect of the wind and ocean currents are neglected, only wave disturbances are considered.

Wave modeling for open sea conditions is a very deep topic. Empirical models can be found in literature from Bretschneider, Pierson-Moskowitz, JONSWAP and
Torsethaugen Spectrum [3]. However, for closed loop systems and control analysis, linear wave response models are developed. In this approach, the empirically determined spectrum is approximated with a linear transfer function with similar power spectrum, driven by Gaussian noise.

Stated mathematically, the wave height \( y(s) \) is

\[
y(s) = h(s)w(s)
\]

where \( w(s) \) is zero mean Gaussian white noise process with unity power across the spectrum \( (\Phi_w(\omega) = 1) \) and \( h(s) \) is a transfer function. The power spectral density function for \( y(s) \) can be written as

\[
\Phi_y = |h(j\omega)|^2
\]

The final 2nd-order linear wave response transfer function approximation with a damping term was introduced by Saelid et al. (1983) as

\[
h(s) = \frac{K_\omega s}{s^2 + 2\lambda \omega_0 + \omega_0^2}
\]

where the gain constant is defined as

\[
K_\omega = 2\lambda \omega_0 \sigma
\]

with \( \sigma \) being the wave intensity, \( \lambda \) the damping coefficient and \( \omega_0 \) the dominating wave frequency. Hence, the power spectral density function of the output \( y(s) \) is written as

\[
\Phi_y = |h(j\omega)|^2 = \frac{4(\lambda \omega_0 \sigma)^2 \omega^2}{(\omega^2 + \omega_0^2)^2 + 4(\lambda \omega_0 \omega)^2}
\]

The parameters \( \lambda \) and \( \sigma \) can be varied to better fit the Pierson-Moskowitz empirical spectrum. In addition to this, a simpler approach can be used. Figure 2 shows the output when white noise is filtered through the 2nd order transfer function and a sine wave of the same dominating frequency \( \omega_0 \) with added stochastic white noise that acts as

\[
y(t) = A\sin(\omega_0 t + \phi) + w(t)
\]

is used instead of the 2nd order transfer function approach. This facilitates the method we use to define the time at which the wave hits each ships based on modifying the phases in the sine wave. For example, if the wave hits TCraft first, there will be phase delay until it reaches the Sea Base. In addition, it is straightforward to use this approximation to orient the ships relative to the wave front by modifying the phases.

2.2. Modeling ship motion due to waves

Figure 2 diagrams the basic shapes and connection of the vessels. We apply the following assumptions of the linear wave theory to this system: the sea water is incompressible, inviscid, there is no surface tension, the fluid motion is irrotational, and the wave amplitude is significantly smaller than the wavelength. Linear wave theory allows us to express the wave model as a superposition of sine waves with differing frequencies. The resulting wave will produce an irregular pattern and is considered a useable approximation to model wave behavior. The ocean waves affect a semi-immersed body by inducing forces that act on the body that will influence its motions. The term used to describe these forces is restoring force [2]. This restoring force arises from the buoyancy and gravity force, which both depend on the waves. Although generally a three dimensional body moves in six DOFs, we will only model the three relevant DOFs that are affected by the hydrostatic force and moments from the waves. The DOFs considered in the model are: roll, pitch and heave. These restoring forces tend to return a floating body to its initial position, while no opposing hydrostatic restoring forces exist in the surge, sway or yaw directions. In other words, since these motions are analogous to a spring-damper system, their respective spring constants and damping coefficients are set to zero and therefore not considered.

An approximation to the equations of motion that govern the roll, pitch and heave take the form of an uncoupled second order dynamical system [2]. The equation for roll motion with linear damping and linear waves is given as
\[
\ddot{\phi} + 2b \dot{\phi} + \omega_k^2 \phi = \omega_p^2 \frac{2\pi \zeta_\phi}{\lambda} \sin(\omega_p t), \quad \omega_p = \sqrt{\frac{gGM_k}{J_k}}, \quad (0.7)
\]
\[
i_s = \frac{J_k}{\Delta}
\]

where \(b\) is a linear damping coefficient, \(\omega_p\) is the ship natural frequency in roll, \(\zeta_\phi\) is the wave amplitude, \(\omega_k\) is the wave angular frequency, \(\overline{GM_k}\) is the metacentric height in roll, \(i_s\) is the mass radius of gyration in roll and equals \(w/(2\sqrt{3})\) where \(w = \) width of half cylinder, \(J_k\) is the mass moment of inertia, \(\Delta\) is the mass displacement from the floating body. The roll equation can be rearranged to

\[
J_k \ddot{\phi} + 2J_k b \dot{\phi} + g\Delta GM_k \phi = g\Delta GM_k \frac{2\pi \zeta_\phi}{\lambda} \sin(\omega_p t) \quad (0.8)
\]

The undamped uncoupled pitch equation is

\[
\dot{\theta} + \omega^2 \dot{\theta} = \omega_p \gamma \sin(\omega_p t), \quad \omega_p = \frac{2\pi}{T_k}, \quad (0.9)
\]

\(T_k = \frac{\lambda}{c - \nu \cos(\alpha)}\), \(\omega_p = \sqrt{g\Delta GM_\theta / i_p}\)

where \(\omega_p\) is the ship natural frequency in pitch, \(\gamma\) is the maximum pitch amplitude, \(\omega_\theta\) is the angular frequency of encounter (number of waves seen by the ship per unit time), \(i_p\) is the mass radius of gyration in pitch and equals \(L/(2\sqrt{3})\) where \(L = \) length, \(v\) is the ship speed, \(c\) is the wave celerity, \(\alpha\) is the angle between ship speed and wave celerity. The pitch equation can be rearranged to yield:

\[
J_\theta \ddot{\theta} + g\Delta GM_\theta \dot{\theta} = g\Delta GM_\theta \gamma \sin(\omega_p t) \quad (0.10)
\]

The uncoupled heave equation is

\[
(m + A_z) \ddot{z} + b \dot{z} + \rho g A_z \dot{z} = \rho g A_z \zeta_o \cos(\omega_h t) \quad (0.11)
\]

where \(A_z\) is the added submerged mass due to the heave, \(A_z\) is the waterplane area of the floating body, which both depend on the waves’ frequency of oscillation.

The two metacentric heights and waterplane area used in the simulations are calculated for half-cylinder models of the two ships. Since no explicit expression for the damping coefficients were available they are estimated in an ad hoc manner provided they comply with intuitive ship motions as a result. Summarizing the resulting spring and damping constants we get

\[
k_{roll} = g\Delta GM_k, \quad k_{pitch} = g\Delta GM_\theta, \quad k_{heave} = \rho g A_z \quad (0.12)
\]

\(b_2 = 2J_k b, \quad b_1 = -b, \quad b = 0.01\)

The above expressions are used in the SimMechanics model to simulate the ocean wave disturbances by its analogous spring damper system.

Figure 4: Basic extremum seeking loop

Although these equations were derived for a single ship, instead of an interconnected ship-ramp-ship system, nevertheless the resulting spring and damper coefficients were used as an estimate for the system’s motion. In reality there exists some coupling between the various motions, which these equations do not capture. For example, the combination of roll and pitch motions will induce yaw and heave motions. Also, during the roll motion, the center of buoyancy will move and cause some pitch motion. It is important to understand the limitations of these equations, which fall short at describing the full behavior of a floating body.

3. EXTREMUM SEEKING OPTIMIZATION

3.1. Background and Implementation

Extremum seeking is a powerful tool that is able to optimize parameters of a non-linear system around a local minimum or maximum. It uses a sinusoidal perturbation to extract gradient information to achieve this goal. Among its beneficial properties is that it is not model based. The motivation behind creating a SimMechanics model becomes evident when implementing this optimization technique, which consists simply of tuning an appropriate loop instead of using explicit model dynamics to find an optimal configuration.

Figure 4 shows the most basic one parameter extremum seeking loop. The output is first filtered through a high pass filter, demodulated, integrated with a gain, and then receives an added perturbation. This estimate is fed back into the system until the output reaches a steady extremum. Rigorous proofs for stability for various systems are given in [1]. Each system is unique, and for the purposes of this environment specific aspects are appropriately altered.

In our model, we use a modified two parameter extremum seeking loop to tune ramp length, \(R\), and ship orientation relative to the wave front, \((\alpha)\), shown in Figure
5. The values of these variables for this particular setup are $h_1 = 0.02, h_2 = 10, l = 1, P = 1.003, \alpha_1 = \alpha_2 = 1.04, k_1 = -1, k_2 = -0.09, \omega_1 = 0.01, \zeta = 30$

Though ES is not model based, it is important to understand the effect of each tuned parameter. While ramp length has an obvious effect on ramp pitch angle, the wave orientation relative to the ships has a less intuitive impact. The goal is to be able to, in real-time, search for an optimal configuration by varying each parameter until an associated cost is minimized. Since the pitch angle decreases nearly monotonically with ramp length, a ramp penalty is imposed to create concavity in the cost map necessary for extremum seeking to function properly while also avoiding extending the ramp to arbitrarily large values. It is also important to note the pitch angle between the ships is an oscillatory signal, therefore we extract the amplitude in order to get a single value for calculating cost.

We accomplish this by filtering the pitch angle for each configuration and multiplying the value by a function of the ramp length. Figure 6 shows two different filtering results, one where all of the oscillatory behavior is eliminated entirely for the cost map (top) and the other allowing some oscillation to be used in the extremum seeking loop (bottom). We repeat this process for varying ramp length of 5 to 20 meters and wave front angles of 0 to 90 degrees to produce a three dimensional cost map that can be analyzed for local minima. This yields the cost map in Figure 7. The cost map creates insight into the impact of each parameter as well as a way to verify the parameter values of the extremum seeking loop.

Some important design criterion included the need to avoid rapid extension and contraction of the ramp and orientation aligning. Extremum seeking requires gradient information that is typically provided through an added sinusoidal perturbation. However, due to the periodic nature of the cost due to wave motion, the signal naturally contains a sinusoidal presence and is not added separately in the ES loop as shown in . Additionally, in order to avoid rapid movement in the model, we filter out the high frequency content only leaving the slower components to actuate the system. These developments are only made possible by the dynamics of the system and only by successfully altering the algorithm are we able to use them to our benefit. The result is smoother actuation that doesn’t jar the system back and forth while still operating in a functional manner.

3.2. ES Results and Discussion

The ships were configured in a bow to stern orientation connected by a pitch jointed ramp. Figure 10 shows the results of a system beginning at a heading angle of 60 degrees and an initial ramp length of 5 meters. The heading output decreases, slows down slightly around 40 degrees, and then continues until slightly overshooting then settling back to 28.5 degrees. The ramp length gently extends and

Figure 5: Modified two parameter extremum seeking loop

Figure 6: The final value from the top is stored to form a cost map while the bottom signal is fed into the ES loop

Figure 7: Three dimensional cost map
The pitch oscillations are effectively reduced from an initial amplitude of 15 degrees to just over 5 degrees from beginning to end. Figure 11 shows the pitch angle evolution with no ES tuning (top) and its reduction while using the ES algorithm (bottom). The results show the favorable attributes we attempted to create while working on a slow enough time scale to be realistically implementable.

We chose the initial condition to allow optimum seeking while avoiding the local minimum in cost along the way. Since large perturbations aren’t present in the output of the signal, it is more difficult to avoid local minima than when using standard ES algorithms. Care must be given when selecting gains and other tuning parameters. In order to avoid these local minima, we suggest beginning seeks from a relatively known position. While ramp length begins at a fixed 5 meters, the vessels should begin sailing into the oncoming waves before beginning the search. From there, small variances in initial positions are acceptable as opposed to beginning perpendicular to the wave front. This allows a gradual decent to the minimum while avoiding the enclosed local minima.

4. PASSIVE CONTROL

4.1. Implementation

The goal of the passive control investigation is to mimic automotive shock absorbers with springs and dampers. This reduces the angle amplitudes of the ramp between the Sea Base and the T-Craft. We investigate three joint cases of the cargo transfer system with these passive control techniques. The three types of joints are: pitch only joint (P-Joint), pitch/roll joint (PR-Joint), and pitch/roll/yaw joint (PRY-Joint). We use the same initial conditions for each simulation. The ramp length is 5 meters and the wave front angle, \( \alpha \), is 45 degrees. We use a damping coefficient of 100 Ns/m in all cases. This magnitude is fixed to provide minimal impact on angle amplitude that instead will vary with spring rates.

In the PRY-Joint case, we introduce springs in the roll and yaw DOFs to stabilize the system. Otherwise, the two ships end up crashing during the simulation. In the PRY-Joint case, the T-Craft’s (TC) roll and yaw spring rates are \( 1 \times 10^2 \) N/m and \( 1 \times 10^6 \) N/m respectively, and the Sea-Base’s...
(SB) roll and yaw rates are $1 \times 10^5$ N/m and $1 \times 10^6$ N/m respectively. We consider these spring rates the minimum values to maintain stability and name them the uncontrolled case. The controlled case refers increasing spring rate beyond the uncontrolled case. In each joint case, we ground the Sea-Base in the surge and sway directions to improve system stability.

We first simulate the system with no spring/damper absorber on either joint in order to get an estimate of the baseline angle amplitude values during a stable limit cycle. This is the open loop response. In this simulation we measure the following variables: pitch, roll, yaw angles, and the difference in heave between the two ships. The difference in heave measurement is calculated using the absolute value of the distance between the center of gravity of the T-Craft and Sea-Base. Due to the ships’ different inertial and buoyancy properties their z-direction position is not necessarily going to be equal. The relative distance between the centers of gravity of the two ships directly relates to the pitch angle oscillations. If the difference in heave decreases in magnitude, the pitch angle amplitude will decrease accordingly. The open loop simulation results for the three cases for the TC and SB joint angles and heave are summarized on the first row of Figure 12, Figure 13 and Figure 14 respectively.

We then simulate the system applying a spring/damper to the TC joint that is analogous to position and velocity feedback. For the pitch only case, we only use one spring/damper pair. We use two for the pitch/roll case and three for the pitch/roll/yaw case, one for each DOF. During the simulations the following measurements are taken: TC joint angles (pitch, roll, and yaw), difference in heave between the two ships and passive control effort (forces from spring/damper). The results are summarized in the second rows of Figure 12, Figure 14 and first row of Figure 15 respectively. In case of absorber failure, the control effort data can be used for the actuator requirements to provide the necessary torque for stability.

Finally, we simulate the system applying a spring/damper to the SB joint. The results of the angles, passive control effort and difference in heave and from the controlled SB joint case are summarized in the second rows of Figure 13, Figure 15 and third row of Figure 14 respectively.


4.2. Passive Control Results

A summary of all the results is given in Figure 12 - Figure 16. Figure 12 summarizes the TC joint maximum angles of the open-loop and controlled system for the three joint cases. Figure 13 summarizes the SB joint maximum angles of the open-loop and controlled system for the three joint cases. Figure 14 summarizes the heave difference for the three cases using no control, TC joint control, and SB joint control. Figure 15 summarizes the passive control effort for the three cases using TC joint control and SB joint control. Figure 16 summarizes the ramp angles for all the cases and absorber arrangements considered. In the bar graphs the abscissa correspond to the three joint cases considered, namely pitch only joint, pitch/roll joint, and pitch/roll/yaw joint respectively and the ordinate the corresponding relevant value.

4.3. Discussion of Joint Angles

The above results summarize the motion at the joints for the various cases mentioned with and without shock absorbers. The open loop simulations show, for each of the cases in both the TC and SB joints, that the pitch angle oscillated between about ±10 degrees. In the PR-Joint case, the TC and SB joints experienced a roll angle oscillation of 0.34 and 0.89 degrees respectively. Any difference in TC and SB angle oscillations may be due to their different dimensions, inertial properties and the Sea-Base being grounded in the surge and sway directions while the T-Craft was not. In the PRY-Joint case, the TC and SB roll angles were 2.93 and 2.76 degrees respectively, and their respective yaw angles were 1.14 and 1.09 degrees. The yaw angle oscillations in both joints are significantly more sensitive. These DOFs have a higher tendency to become unstable, and therefore require larger spring rate in the joints for stabilization. Increasing the spring rate in the yaw DOF decreases its influence on the behavior of the system, hence closely mirroring the PR-Joint case, which has a desired stable system response. Furthermore, increasing the spring rate of the roll DOF makes the system response closely mirror the P-Joint case, which also has a stable response.

The heave summary plots in Figure 14 show the improved performance between the open and closed loop simulations. With no absorbers the average delta heaves between the T-Craft and Sea-Base were 3.89 m, 4.01 m, and 4.00 m for the P-Joint, PR-Joint, and PRY-Joint cases respectively; for an absorber on the TC joint the delta heaves were 3.03 m, 3.07 m, 3.07 m for each case respectively; for an absorber on the SB joint the delta heaves were 3.02 m, 3.07 m, 3.07 m for each case respectively. The important point to see from these results is the decrease of about 1 m in delta heave between the uncontrolled and controlled simulations, which primarily affects the pitch angle, reducing it a few degrees.

The results of the controlled simulations show an improvement in the amplitude of angle oscillations for each of the DOFs as shown in Figure 12 and Figure 13. The amount of reduction depends on the spring constant chosen. An arbitrary constant was chosen in the effective order of magnitude range for the simulations to show a reduction in angle amplitudes. For all three controlled cases the pitch angle was reduced to about ±0.10 degrees for both the TC and SB joints with a gain value of $K = -5 \times 10^8$. In the controlled PR-Joint case the roll angle oscillation reduced to about ±0.21 and ± 0.59 degrees for the TC and SB joints respectively with a gain value of $K = -1 \times 10^9$. The smaller oscillation was ±0.06 and ± 0.10 degrees for the TC and SB joints respectively with a gain value of $K = -1 \times 10^8$, and the yaw angle oscillation was ±0.10 and ± 0.22 degrees for the TC and SB joints respectively with a gain value of $K = -1 \times 10^8$. Comparing the PR-Joint to the PRY-Joint we see that the addition of the yaw DOF significantly influences the gain tuning by a few orders of magnitude to reach a similar steady state value for the roll angle in the PR-Joint case. As assumed the complexity and unpredictability of the response grows with the amount of DOFs.

4.4. Discussion of Ramp Angles

Another relevant set of values to look at is the ramp angles since the above joint data misses some crucial information. For example, in each of the joint cases, the only measured angle is the allowed DOF(s). In the pitch joint case, only the pitch angle is measured while the roll and yaw angle amplitudes of the ramp are unmeasured. If the ramp’s roll DOF is unstable, it can be considered unobservable in the above joint measurements and would not be known if we only rely on the joint angle measurements. Measuring the ramp angles directly eliminates this problem. Figure 16 summarizes the ramp angles for each of the three cases and for four various scenarios, namely: no absorbers used in joints, an absorber on the T-Craft joint, an absorber on the Sea Base joint, and an absorber on both joints. Matching our intuition, the ramp
angles are mostly maximal when no absorbers are used (with two exceptions as seen in figure), and mostly minimal when both joints have absorbers. The addition of each DOF creates unintuitive reactions in angle amplitudes. For example, when no absorber is used the roll angle is minimum in the PR-Joint case and greater in the P- and PRY-Joint case for no apparent reason. A future endeavor is to tune the dampers using ES in order to reduce the angles amplitudes even further.

5. FIN CONTROL

5.1. Control Plant Model

In this section the ability of actuating the smaller vessel with fins in the pitch DOF to reduce the roll of the ramp is considered. A simplified version of the system is simulated, and an estimate for the necessary size of the fins is provided. Preliminary analysis has shown that forcing even the T-Craft in heave will require a prohibitive amount of force; therefore the control task is limited to actuating the smaller ship in pitch. To provide the fins ample space, only the bow-to-stern configuration is considered. In order to maximize the torque per area unit of the fins, four fins were proposed on each side of the ship, with one pair close to the bow and another pair close to the stern.

In hydrodynamic application, it is usual to separate between the Process Plant Model (PPM) and the Control Plant Model (CPM). The first one, being more complex, intends to incorporate everything known about the plant dynamics and is used to make high-fidelity simulation. Conversely, a CPM needs to be a simple model, only describing the most important dynamics of the system for use in the controller.

Thus, before we design the controller, we simplify the dynamics. Neither of the ships’ movement in roll, sway and yaw affects the roll of the ramp. For this reason, only pitch, heave and surge are modeled in this section, reducing the model to two dimensions. Secondly, we linearize the dynamics. The resulting model is expressed using Hamiltonian Mechanics.

The state variables $x_1$, $z_1$ and $\theta_1$ for the Seabase are illustrated on Figures 17 and 18. Variables $x_2$, $z_2$ and $\theta_2$ for the T-craft are defined similarly. Defining state vector

$$x = [x_1, z_1, \theta_1, x_2, z_2, \theta_2]^T$$

This vector has six dimensions, while the system has only five degrees of freedom. We select $x_2$ to be expressed with other variables and define vectors

$$q = [x_1, z_1, \theta_1, z_2, \theta_2]^T$$

Illustrated in Figures 17 and 18 along with

$$\nu = \frac{d}{dt}q = [\dot{x}_1, \dot{z}_1, \dot{\theta}_1, \dot{z}_2, \dot{\theta}_2]^T$$

The kinetic energy can be expressed as

$$T(\nu) = \frac{1}{2} \nu^T M \nu, \quad M = M^T > 0 \in \mathbb{R}^{5 \times 5}$$

where the conjugate momenta $p = \partial T / \partial \nu$ is $p = Mp \in \mathbb{R}^5$, yielding

$$T(p) = \frac{1}{2} p^T M^{-1} p$$

Assuming linear box-shaped vessel (same as [4] section 3.2.3), restoring force potential can be expressed as

$$V(q) = \frac{1}{2} L_1 W_1 \rho_\text{sw} g z_1^2 + \frac{1}{6} \rho_\text{sw} g W_1 L_1 \theta_1^2$$

$$+ \frac{1}{2} L_2 W_2 \rho_\text{sw} g z_2^2 + \frac{1}{6} \rho_\text{sw} g W_2 L_2 \theta_2^2$$

with the Hamiltonian being $H(p,q) = T(p) + V(q)$, and equations of motion

$$\dot{p}_j = -\frac{\partial H(p,q)}{\partial q_j} + Q_j$$

$$\dot{q}_j = \frac{\partial H(p,q)}{\partial p_j}$$

for each DOF $j$. The generalized force $Q_j$ is defined

$$Q_j = \sum_i F_i \frac{\partial x_i}{\partial q_j}$$

Figure 19: Visualization taken during fin simulation
where $x_i$ is a coordinate vector with $i$ elements and is used to include the forces resulting from damping as well as control inputs, which are not derivable from a potential.

Hydrodynamic force is considered to be dependent on velocity and acceleration of the vessel [4]. These forces are linearized as follows:

$$\{ F_M, F_H, F_A \} = -M_A \ddot{v} + D \dot{v}$$  \hspace{1cm} (0.21)

with $M_A$ typically referred to as added mass that is generated by the inertia of displaced water caused by the body moving through the liquid. Again referring to [4], $M_A$, unlike the mass tensor in (0.16), is generally not symmetric but the associated kinetic energy is

$$T_{M_A} = \frac{1}{4} v^T (M_A + M_A^T) v$$  \hspace{1cm} (0.22)

which is non-negative. The term $\frac{1}{2}(M_A + M_A^T)$ can be added to the regular rigid body mass tensor in (0.16), thus justifying the terminology.

Damping $Dv$ is calculated as a generalized force in coordinate system defined by (0.14).

Numerical values for the simulation are used from non-dimensional models from the GNC toolbox, using the length of the Sea Base and T-Craft (200m and 40m respectively) to calculate the dimensionalized values in particular the Mariner class vessel. A snap shot of the simulation is shown in Figure 19.

5.2. Control Input and Controller

The control in pitch is performed by configuring the fins in front and in the back of the T-craft to generate forces in opposite directions. As far as the controller is concerned, the required torque was the only input needed. We also calculate the size of the fins needed to generate the required torque and discuss it further on.

The controller itself is an infinite horizon continuous time LQR. The most important cost function to be minimized is $(z_2 - z_1)^2$, since $z_2 - z_1$ is a diffeomorphism on the roll angle of the ramp. This requires a transformation of the state space equations so as to include $z_2 - z_1$ in the state space. Both the transformation and implementation of LQR are done using Matlab and are not discussed in further detail.

5.3. Fin Control Results

Behavior of the two-ship system without any actuation is illustrated on Figure 20. The line indicates the difference between heights of attachment points $(z_2 - z_1)$. The maximal difference is approximately 2.2 meters. Since the waves are modeled as noise-driven linear filter, it takes some time before the wave model is fully excited. Figure 21 shows the fin actuated system and the subsequent reduction in ramp roll.

Figure 22 illustrates the actuation torque required to achieve the reduction. The maximal torque requirement is $60 \text{MNm}$. Since the T-craft is 40 meters long, the fins can maximally be placed 20 meters away from the center of
gravity. Thus, the total force the fins provide along the length of the T-craft is \(3 \text{MN}\).

The required fin area at a given speed is calculated using standard formulae from either aerodynamic or hydrodynamic literature. The relationship between kinematic viscosity of water and air means that water flow around a fin at 10 m/s is similar to air flow around a wing at about 100 m/s, the exact number being temperature-dependent. This means that aerodynamic literature such as [6] can be applied directly. The non-dimensional lift coefficient is defined as

\[
C_L = \frac{L}{0.5 \rho v^2 A}
\]

where \(L\) is the lift force, \(\rho\) is the density of the working fluid, \(v\) is the speed and \(A\) is the area of the fins. From Table 7.10 in [6] it may be seen that the lift coefficient is about 1.2 in for a foil with aspect ratio of six. Substituting \(\rho = 1030 \text{kg/m}^3\) and \(v = 10 \text{m/s}\), yields \(58 \text{m}^2\). This area can be provided with four fins of \(1.6 \times 9.3\) m each, a feasible size.

Although the estimation work had to be rough due the system being in conceptual stage, the results achieved suggest that a fin-actuated T-craft can be built yielding significant boosts in overall system performance.

6. CONCLUSIONS AND FUTURE WORK

In this paper, pitch, roll and heave motions of two ships with a ramp connection were modeled as a series of rigid bodies acted on by a wave force model analogous to a spring-damper system. Constants for the springs and dampers were derived and the model was implemented in SimMechanics for simulation. First, we optimized the ship orientation relative to the wave heading angle and ramp length by extremum seeking. The optimal values for heading angle and ramp length were correctly identified as 28.5 degrees and 11.5 meter, respectively. Secondly, we investigated passive control techniques to stabilize the ramp making cargo transfer easier. Three different joint cases were simulated and the results show that putting an absorber on both joints provides the most reduction in angle amplitude. Finally, an active control approach was investigated using fin control to reduce the relative movement of the connection points between the ramp and each of the ships. This method showed a great reduction in pitch amplitude of the T-craft, which greatly reduced the pitch amplitude of the ramp.

We did not present a systematic way to choose the absorber constants for passive control technique. In future research we will optimize the spring constants and damper coefficients to achieve the desired angle amplitude reduction. In addition, roll stabilization techniques such as an anti-roll tank, which is based on a Frahm vibration absorber, will be incorporated into this research. This idea utilizes port to starboard transfer of water out of sync with the roll of the ship. A modification of this technique for pitch stabilization during the cargo transfer between the two ships will also be considered in future work. Finally, these methods are being tested on a time domain sea keeping program that more accurately models the dynamics of the system. This will help to verify the results in a more rigorous simulation environment.

7. REFERENCES